DE-FG05-92ER40717-12

The $16\frac{1}{2} - N_f$ Expansion and the Infrared Fixed Point in Perturbative QCD

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Abstract:

In QCD with $16\frac{1}{2} - \epsilon$ massless quark flavours there is an infrared fixed point with $\alpha_s/\pi = \frac{8}{321}\epsilon$, in the limit $\epsilon \to 0_+$. I develop the idea of Banks and Zaks to expand about $N_f = 16\frac{1}{2}$. This expansion is certainly useful for $N_f = 16, 15, 14, \ldots$, and arguably it can reach the phenomenologically interesting case $N_f = 2$, where it suggests that α_s/π "freezes" to a value of order of magnitude 0.4 in the infrared.

1. In QCD with N_f massless flavours asymptotic freedom is lost if N_f exceeds $16\frac{1}{2}$, which is where the first coefficient of the β function vanishes. In my notation β has the form:

$$\beta(a) \equiv \mu \frac{da}{d\mu} = -ba^2(1 + ca + c_2 a^2 + \ldots), \tag{1}$$

where $a \equiv \alpha_s/\pi$, $b = (33 - 2N_f)/6$, and $bc = (153 - 19N_f)/12$. Since b has been factored out, $c, c_2, \ldots$ will naturally each have a simple pole at $N_f = 16\frac{1}{2}$. For N_f just below $16\frac{1}{2}$ there is a zero of the β function at $a^* \approx -1/c$, a small positive value. If the couplant $a(\mu)$ lies between 0 and a^* at some energy scale μ, then it is trapped in this range at all energies. One then has weak coupling at all energies [1, 2].

In 1982 Banks and Zaks [2] suggested an expansion in powers of $16\frac{1}{2} - N_f$. The idea has since been used by White [3] to study the Pomeron in QCD. Grunberg [4] (though in a context I deplore) has discovered some important features of the expansion. I argue here that this expansion is relevant to the real world with just two light flavours. It implies that, as a *perturbative* effect, the QCD couplant "freezes" in the infrared. This corroborates Ref. [5], which studied $R_{e^+e^-}$ in third-order "optimized" perturbation theory [6].

Readers who find this hard to swallow should nevertheless read on: This expansion is fun and it certainly offers insight into theories with $16, 15, 14, \ldots$ flavours, whether or not one believes it is directly useful for $N_f = 2$.

The following analysis stays entirely within the realm of perturbation theory, and I shall discuss nonperturbative effects only briefly at the end.

2. The natural expansion parameter for the Banks-Zaks (BZ) expansion is

$$a_0 \equiv \frac{8}{321}(16\tfrac{1}{2} - N_f), \tag{2}$$

which is the limiting form of $a^* \sim -1/c$ as $N_f \to 16\frac{1}{2}$ from below. Because the coefficient $8/321$ is so small, a_0 is tiny ($\sim 1/80$) for $N_f = 16$ and remains of modest size (0.36) even for $N_f = 2$.

It is convenient to re-write perturbative coefficients, eliminating N_f in favour of a_0 [4]. The first two β-function coefficients are:

$$b = \frac{107}{8} a_0, \qquad c = -\frac{1}{a_0} + c_{1,0}, \tag{3}$$

with $c_{1,0} = 19/4$. The higher-order β-function coefficients are renormalization-scheme (RS) dependent. In any 'regular' scheme their N_f dependence is such that one may write them as [7, 4]:

$$c_2 = \frac{1}{a_0}\left(c_{2,-1} + c_{2,0}a_0 + c_{2,1}a_0^2 + c_{2,2}a_0^3\right), \tag{4}$$

and so on. (Note that a term $c_{i,j}a_0^p$ can be assigned a degree $i+j-p$, and all terms in any formula have the same degree.) In the $\overline{\rm MS}$ scheme one has [8]:

$$c_2(\overline{\rm MS}) = \frac{1}{a_0}\left(-\frac{37117}{10272}+\frac{243}{32}a_0+\frac{34775}{1536}a_0^2\right). \tag{5}$$

Consider some perturbatively calculable physical quantity in QCD. The prototypical example is $R_{e^+e^-}(Q) = 3\sum q_i^2(1+\mathcal{R})$, where

$$\mathcal{R} = a(1+r_1a+r_2a^2+\ldots). \tag{6}$$

In any 'regular' RS the coefficients r_i are ith-order polynomials in N_f, and hence in a_0:

$$r_1 = r_{1,0}+r_{1,1}a_0, \tag{7}$$

$$r_2 = r_{2,0}+r_{2,1}a_0+r_{2,2}a_0^2, \tag{8}$$

etc.. All these coefficients are RS dependent. In the $\overline{\rm MS}$ scheme for the $R_{e^+e^-}$ case one has [9]

$$r_1(\overline{\rm MS}(\mu=Q)) = \frac{1}{12}+\frac{107}{32}(11-8\zeta(3))a_0, \tag{9}$$

$$r_2(\overline{\rm MS}(\mu=Q)) = \left[-\frac{12521}{288}+13\zeta(3)\right]+\mathcal{O}(a_0). \tag{10}$$

It is noteworthy that $\zeta(3)$ does not appear in $r_{1,0}$ and $\zeta(5)$ does not appear in $r_{2,0}$. (I have ignored the $(\sum q)^2$ term in r_2; its N_f dependence depends on the electric charges assigned to the ficticious extra quarks.)

3. Consider first the BZ expansion for $\mathcal{R}^* \equiv \mathcal{R}(Q\to 0)$. One first solves the fixed-point condition $\beta(a^*)=0$ for a^* as a series in a_0. One then substitutes in Eq. (6), again expanding in powers of a_0. To leading order $\mathcal{R}^* = a^* = a_0$, while at second order one has [4]

$$a^* = a_0[1+(c_{2,-1}+c_{1,0})a_0+\mathcal{O}(a_0^2)], \tag{11}$$

and hence

$$\mathcal{R}^* = a_0[1+(r_{1,0}+c_{2,-1}+c_{1,0})a_0+\mathcal{O}(a_0^2)]. \tag{12}$$

The sum of $r_{1,0}$ and $c_{2,-1}$ is RS invariant, as I show later. ($c_{1,0}=19/4$ is invariant.) In the e^+e^- case one obtains $\mathcal{R}^* = a_0(1+1.22a_0+\ldots)$, so the correction is relatively modest. At $N_f=2$ the correction is about 44%. While one cannot give too much credence to the quantitative result, the qualitative message of leading order remains; there seems to be a fixed point of modest size. Figure 1 shows $\mathcal{R}^*$ as a function of N_f and compares first- and second-order BZ results with the OPT results from Ref. [5].

The crucial test of this interpretation will come at next order. A straightforward calculation yields the coefficient of the next order term in (12), which is

$$(c_{1,0} + 2c_{2,-1} + 2r_{1,0})\,(c_{1,0} + c_{2,-1}) + r_{1,1} + r_{2,0} + c_{2,0} + c_{3,-1}. \tag{13}$$

In the e^+e^- case this reduces to $-18.25 + c_{3,-1}(\overline{\rm MS})$. For the expansion to be credible one needs $c_{3,-1}(\overline{\rm MS})$ to be in the range, say, +13 to +21. I expect $c_{3,-1}$ to be found in the lower end of this range, thereby reducing $\mathcal{R}^*$, and bringing it into better agreement with the OPT results. A calculation of the 4th-order β function coefficient would test this prediction.

Note that nth order in the BZ expansion requires $n+1$ terms in the β function, but only n terms in $\mathcal{R}$. Thus, in terms of diagrammatic information used, it is intermediate between nth and $(n+1)$th order of ordinary perturbation theory.

The coefficients in the BZ expansion of $\mathcal{R}^*$ are RS invariant. One can prove this by considering the RS invariants ρ_i [6] and expanding them in powers of a_0. From the leading $1/a_0$ term in $\tilde{\rho_2} \equiv \rho_2 + c^2/4 \equiv r_2 + c_2 - r_1^2 - cr_1$ one sees that $r_{1,0} + c_{2,-1}$ is invariant, as claimed earlier. From the subleading part of $\tilde{\rho_2}$ one finds that $r_{2,0}+c_{2,0}-r_{1,0}^2+r_{1,1}-c_{1,0}r_{1,0}$ is invariant. Then from the leading $1/a_0$ term in $\rho_3 \equiv r_3 + \frac{1}{2}c_3 - r_1(c_2 + 3r_2 - 2r_1^2 - \frac{1}{2}cr_1)$ one finds that $c_{3,-1} - 2r_{1,0}c_{2,-1} - r_{1,0}^2$ is invariant. It is then straightforward to show that the combination in Eq. (13) is RS invariant. It also follows that $c_{2,-1}^2 + c_{3,-1}$ is invariant [4].

4. Next, consider a BZ expansion for $\mathcal{R}$ at a general Q. There are three preliminary steps. Firstly, one integrates the β-function equation to obtain:

$$b\ln(\mu/\tilde{\Lambda}) = \lim_{\delta\to 0}\left[\int_\delta^a \frac{dx}{\hat{\beta}(x)} + \mathcal{C}(\delta)\right], \tag{14}$$

where $\hat{\beta}(x) \equiv \beta(x)/b$, and $\tilde{\Lambda}$ is a constant with dimensions of mass. The constant of integration, $\mathcal{C}(\delta)$, must be suitably singular in the limit $\delta \to 0$, and I choose [6]

$$\begin{aligned} \mathcal{C}(\delta) &= \boldsymbol{P}\int_\delta^\infty \frac{dx}{x^2(1+cx)}, \\ &= \frac{1}{\delta} + c\ln\delta + c\ln|c| + \mathcal{O}(\delta), \end{aligned} \tag{15}$$

where $\boldsymbol{P}$ (principal value) is specified because of the pole at $x = -1/c$ when c is negative. This choice amounts to a definition, in a general RS, of the $\tilde{\Lambda}$ parameter. The commonly used Λ parameter [10] is defined in a less natural way, and is related by an RS-invariant,

but N_f-dependent factor; $\ln(\Lambda/\tilde{\Lambda}) = (c/b)\ln(2 \mid c \mid /b)$. The two Λ's become infinitely different in the limit $N_f \to 16\frac{1}{2}$.

Secondly, recall that for each physical quantity $\mathcal{R}$ there is an RS invariant [6]:

$$\rho_1 \equiv b\ln(\mu/\tilde{\Lambda}) - r_1. \tag{16}$$

The renormalization scale μ cancels out because the coefficient r_1 always contains a $b\ln(\mu/Q)$ piece. Furthermore, the $\tilde{\Lambda}$ parameter is scheme dependent in a way that exactly cancels the scheme dependence of the remaining part of r_1 [11, 6]. (ρ_1 must be regarded *as a whole*; splitting it into pieces spoils RS invariance. It *cannot* be written as $A + Ba_0$ with A and B being RS invariant [12]. The reason is the $1/b$ factor in the RS transformation of $\tilde{\Lambda}$: For two schemes related by $a' = a(1 + v_1 a + \ldots)$ one has $\ln(\tilde{\Lambda}'/\tilde{\Lambda}) = v_1/b$ [11]. For 'regular' schemes v_1 is linear in N_f, but is otherwise arbitrary. The other ρ_i invariants *can* be split into different orders in a_0 because they are not $Q/\tilde{\Lambda}$ dependent.) One may think of ρ_1 as $b\ln Q/\tilde{\Lambda}_{\rm eff}$, where $\tilde{\Lambda}_{\rm eff} \equiv \tilde{\Lambda}\exp(r_1/b)$ is a scale specific to the particular physical quantity $\mathcal{R}$. Each $\tilde{\Lambda}_{\rm eff}$ can be related in an exactly known way, once the corresponding r_1 has been calculated, to the universal $\tilde{\Lambda}$ of some reference scheme (say, $\overline{\rm MS}$), which plays the role of the single free parameter of the theory.

Thirdly, let us reconnoitre the BZ limit. Since a is trapped between 0 and a^*, it is at most of order a_0. The first two terms in the β function dominate, so Eq. (14) gives

$$b\ln(\mu/\tilde{\Lambda}) = \frac{1}{a} + c\ln\left|\frac{ca}{1+ca}\right| + \mathcal{O}(c_2 a). \tag{17}$$

Combining the two last equations yields ρ_1 in terms of a. The dominant terms are of order $1/a_0$, so one may discard the $\mathcal{O}(c_2 a)$ and r_1 terms, which are of order unity. Since in the BZ limit $c \sim -1/a_0$, $\mathcal{R} \sim a$ and $\mathcal{R}^* \sim a^* \sim a_0$, the limiting form is:

$$\rho_1 = \frac{1}{\mathcal{R}} + \frac{1}{\mathcal{R}^*}\ln\left(\frac{\mathcal{R}^* - \mathcal{R}}{\mathcal{R}}\right). \tag{18}$$

Inverting this equation (numerically) would give $\mathcal{R}$ as a function of $\rho_1 = b\ln Q/\tilde{\Lambda}_{\rm eff}$, and hence as a function of Q. The resulting function $\mathcal{R}(Q)$ is RS invariant, and is universal, except that $\tilde{\Lambda}_{\rm eff}$ depends on the specific physical quantity considered. $\mathcal{R}(Q)$ exhibits both asymptotic-freedom as $Q \to \infty$ and "freezing" behaviour, $\mathcal{R}(Q) \to \mathcal{R}^*$, as $Q \to 0$.

The formulation of a BZ expansion for $\mathcal{R}$ at a general Q is not a completely unambiguous matter. $\mathcal{R}(Q)$ is not expressible as a simple power series in a_0, so some thought is required in deciding how precisely to define the nth-order approximant. The important point, as with any approximation, is to reconcile and make best use of all available information. Simply integrating the β-function equation and then expanding in powers of

a_0 produces correction terms with $(a_0 - \mathcal{R})$ denominators. Higher orders bring in ever more singular terms. However, these terms simply arise from an expansion of $\ln(\mathcal{R}^* - \mathcal{R})$, reflecting the fact that the fixed point $\mathcal{R}^*$ does not stay at a_0, but is itself a series in a_0. Therefore it is sensible to organize the expansion to reflect this.

Thus, before performing the integration in (14), I first re-write the $1/\hat{\beta}(x)$ integrand:

$$\frac{1}{-x^2(1 + cx + c_2x^2 + \ldots)} = \frac{-a^*}{x^2(a^* - x)P(x)}, \tag{19}$$

ensuring that the pole is in the right place. Next, I express it in partial fractions:

$$\frac{1}{\hat{\beta}(x)} = -\frac{1}{x^2} + \frac{c}{x} - \frac{1}{\hat{\gamma}^*(a^* - x)} + H(x). \tag{20}$$

The coefficients of the first three terms are determined by the $x \to 0$ and $x \to a^*$ limits. Hence, $\hat{\gamma}^*$ is γ^*/b, where γ^* is the slope of the β function at the fixed point:

$$\gamma^* \equiv \left.\frac{d\beta(x)}{dx}\right|_{x=a^*} = -ba^*(1 + 2ca^* + 3c_2{a^*}^2 + \ldots). \tag{21}$$

The remainder term can be expanded as a power series, $H(x) = H_0 + H_1x + \ldots$.

In nth order of the BZ expansion one may truncate the β function after $n + 1$ terms. In that case $H(x) = Q(x)/P(x)$, where $P(x)$ and $Q(x)$ are polynomials of degree $n - 1$ and $n - 2$, respectively. (For $n = 1$, $Q(x)$ vanishes.) The coefficients of $P(x)$ are of order unity as $a_0 \to 0$. The coefficients of $Q(x)$ are of order a_0 because of cancellations that make both $c + 1/\hat{\gamma}^*$ and $a^*/\hat{\gamma}^* - 1$ of order a_0. Thus, $H(x)$ has coefficients of order a_0. [H_0, for instance, is $a_0(c_{4,-1} + 2c_{2,-1}c_{3,-1} + c_{2,-1}^3) + \mathcal{O}(a_0^2)$.] In nth order ($n \geq 4$) of the BZ expansion one needs coefficients up to H_{n-4}: for the first three orders one can drop $H(x)$ altogether.

It is now simple to perform the integration in (14) and use (15) to obtain ρ_1 in terms of a and a^*. One may then eliminate a and a^* in favour of $\mathcal{R}$ and $\mathcal{R}^*$, working to the appropriate order. This last step can be short circuited by noting that the final result must be RS invariant, and so, without loss of generality, one may choose to work in the RS in which $\mathcal{R} = a$. Thus, the result in nth order of the BZ expansion can be expressed as:

$$\rho_1 = \frac{1}{\mathcal{R}} + \frac{1}{\hat{\gamma}^{*(n)}} \ln\left(1 - \frac{\mathcal{R}}{\mathcal{R}^{*(n)}}\right) + c \ln(|\, c\,|\, \mathcal{R}) + \sum_{i=0}^{n-4} \frac{H_i \mathcal{R}^{i+1}}{(i+1)}, \tag{22}$$

where $\mathcal{R}^{*(n)}$ and $\hat{\gamma}^{*(n)}$ are the nth-order approximations to $\mathcal{R}^*$ and $\hat{\gamma}^*$, respectively. For small $\mathcal{R}$ (i.e., at large Q) this formula will agree with $(n+1)$th-order perturbation theory to the appropriate order in $\mathcal{R}$ and a_0. For $\hat{\gamma}^*$ a straightforward calculation gives

$$\hat{\gamma}^* = \frac{\gamma^*}{b} = a_0(1 + c_{1,0}a_0 + (c_{1,0}^2 - c_{2,-1}^2 - c_{3,-1})a_0^2 + \mathcal{O}(a_0^3)). \tag{23}$$

γ^* is the 'critical exponent' that governs the manner in which $\mathcal{R}$ approaches $\mathcal{R}^*$ in the $Q \to 0$ limit [13]; $\mathcal{R}^* - \mathcal{R} \sim \text{const.}\, Q^{\gamma^*}$. It should therefore be RS invariant. Actually, this statement must be qualified [13, 14]; γ^* is invariant within the sub-class of RS's whose relation to the $a = \mathcal{R}$ scheme is not singular at the fixed point. The coefficients in (23) are the "universal" invariants (universal in that they are independent of the particular quantity $\mathcal{R}$) discovered by Grunberg [4].

Results for $\mathcal{R}(Q)$ in first and second order are shown in Figs. 2 and 3. One could also obtain hypothetical third-order results by guessing a value for $c_{3,-1}$. For a value around 15, for instance, the $Q = 0$ values are similar to the first-order results, with the shape of the curves being more like second order.

The first-order results use (22) which, unlike the earlier form (18), retains the whole of c in the $c \ln(|\, c\,|\, \mathcal{R})$ term. This seems sensible because it avoids spoiling the behaviour at large Q, known from ordinary perturbation theory. One must know c in order to obtain even the leading-order BZ result, in any case. Another point is that (22) puts the corrections to the coefficient of $\ln(1 - \mathcal{R}/\mathcal{R}^*)$ in the $\hat{\gamma}^*$ denominator, rather than re-expanding them into the numerator. This is sensible because one knows that γ^*, the slope of $\beta(a)$ at a^*, must be positive, as is obvious from a sketch of the β function.

For $N_f = 16, 15, 14, \ldots$ the theory becomes "almost scale invariant." $\mathcal{R}$ remains constant over a huge range of Q about $\tilde{\Lambda}_{\text{eff}}$. This is because $\rho_1 = (107/8)a_0 \ln(Q/\tilde{\Lambda}_{\text{eff}})$ remains close to zero. Only when $Q/\tilde{\Lambda}_{\text{eff}}$ becomes extremely small does $\mathcal{R}(Q)$ abruptly rise up to $\mathcal{R}^*$, while only when $Q/\tilde{\Lambda}_{\text{eff}}$ becomes very large does $\mathcal{R}(Q)$ slowly decrease, as required by asymptotic freedom. The constant value around $\tilde{\Lambda}_{\text{eff}}$ is, in the BZ limit, $\mathcal{R}^*/(1+\chi) \sim 0.78\mathcal{R}^*$, where $\ln\chi + \chi + 1 = 0$. The region over which $\mathcal{R}$ stays within 10% of this value is roughly for $Q/\tilde{\Lambda}_{\text{eff}}$ between the two extremes $\exp(\pm 0.04/a_0^2)$. Thus, this region is very extensive for $a_0 < 0.1$ ($N_f \geq 13$) and is noticeable up to $a_0 \sim 0.2$ ($N_f \sim 9$). For smaller N_f the region $Q \sim \tilde{\Lambda}_{\text{eff}}$ becomes, on the contrary, a region of rapid variation of $\mathcal{R}(Q)$: asymptotic freedom sets in quickly above $\tilde{\Lambda}_{\text{eff}}$, while infrared "freeze-out" occurs just below $\tilde{\Lambda}_{\text{eff}}$. The cause of this different behaviour is the much larger value of the critical exponent γ^* at smaller N_f.

5. The above analysis has been entirely within a perturbative framework.[1] Its main message is that the couplant "freezes" at low energies as a *perturbative* effect. This property is manifest for N_f close to $16\frac{1}{2}$ and the BZ expansion implies that it extends to

[1] The BZ limit might also be a useful way to explore nonperturbative effects, such as instantons, in a controlled weak-coupling context.

all lower values of N_f, albeit with diminishing accuracy. Because of "freezing" one obtains finite perturbative predictions at low energies, without resort to such notions as constituent-quark masses, effective gluon masses, condensates, etc.. I do not claim that these predictions are *right* — nonperturbative effects certainly do exist at low energies — but I do argue that these predictions are *meaningful* and have some predictive power.

Let me first remark that "freezing" is not incompatible with confinement: there is no evidence that confinement necessarily requires the couplant to become infinite in the infrared. (Gribov's ideas [15], for instance, explicitly involve a "freezing" of the couplant.) Confinement and chiral-symmetry-breaking phenomena are associated with terms invisible to perturbation theory, such as $\exp(-1/ba)$, whose Taylor expansion is $0 + 0 + 0 + \ldots$. Even in the $N_f = 16$ theory, where the couplant is always very small, perturbation theory is probably not the whole story. Nonperturbative effects in that case may be extremely tiny but they could still be qualitatively decisive in providing confinement and chiral symmetry breaking. (However, if the confinement radius is enormous relative to any feasible experiment, then the physicists of an $N_f = 16$ world might well regard confinement as an irrelevant notion.)

In the real world there are certainly nonperturbative effects of crucial importance at low energies. The perturbative predictions are smooth functions, whereas the data is characterized by a fine structure of hadronic thresholds and resonances. However, it is natural to conjecture that data and perturbative theory can be compared if some suitable 'smearing' procedure is used to 'average out' the nonperturbative effects. This is an old idea (e.g. [16]) that has appeared in many forms. I am only proposing that the game can be extended right down to zero energy. Indeed, such a comparison with $R_{e^+e^-}$ data proves to be very successful [5]. The hypothesis is worth pursuing because it potentially enlarges our predictive power. It also changes the nature of the debate about how perturbative and nonperturbative effects fit together: The question is no longer "How must we modify perturbation theory to prevent it giving nonsense for $Q \leq \tilde{\Lambda}$?" but instead becomes "How do nonperturbative effects modulate the smooth perturbative result at low energies to produce fine structure?"

Quark masses were ignored in the preceding analysis. However, it should be straightforward to include running masses in the usual way [17]. The c, b, t quarks clearly decouple at energies below the $\tilde{\Lambda}$ scale; the s quark is a borderline case; but it seems reasonable to treat the u, d quarks as approximately massless: Their running masses are of order 5–10 MeV at a renormalization scale of 1 GeV [18], and although the running masses increase

at lower scales this rise is tempered by the freezing of a. In any case, the BZ-expansion results are rather insensitive to whether the effective N_f for $Q \leq \tilde{\Lambda}$ is taken to be 3 or 2 or 0. (It would not be appropriate to invoke constituent quark masses, which have no well-defined connection with the parameters of the underlying Lagrangian [18]. They represent a phenomenological attempt to parametrize some of the nonperturbative effects.)

"Freezing" has long been a popular, *ad hoc* hypothesis, and it has been invoked in a great variety of successful phenomenology. The low-energy values for α_s/π so obtained lie, with remarkable consistency, in the range 0.2 to 0.3. (See Ref. [5] for a brief literature survey.) Usually "freezing" is blamed on unspecified "non-perturbative effects." The BZ and OPT results imply, however, that "freezing" is a phenomenon present in perturbation theory at second or third order. The BZ expansion implies that "freezing" is not attributable to π^2 terms; these only appear in the $\mathcal{O}(a_0^2)$ part of the r_2 coefficient. It also implies that "freezing" is universal, regardless of spacelike/timelike Q^2, and should occur for any perturbatively expandable physical quantity.

Acknowledgements

I am grateful to Alan White for introducing me to the concept of the $16\frac{1}{2} - N_f$ expansion. I thank Jiří Chýla and Andrei Kataev for correspondence, and Frank Close, Poul Damgaard, Valery Khoze, Alan Mattingly, Chris Maxwell, and Graham Shore for discussions.

This work was supported in part by the U.S. Department of Energy under Grant No. DE-FG05-92ER40717.

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